## Unbounded Population MO-CMA-ES for the Bi-Objective BBOB Test Suite

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## UP-MO-CMA-ES in a nutshell

- Population $S$ of Individuals: $\left(x_{i}, \sigma_{i}, C_{i}\right), i=1, \ldots$
while Stopping criterion is not met do
Select parent from $S$ based on Hypervolume Contribution; Sample Offspring with Crossover; if Offspring is non-dominated in $S$ then Adapt $(\sigma, C)$ of parent and offspring; Add offspring to $S$; end else

Adapt $\sigma$ of parent;
end
end

## Parent Selection

- Select parent based on Hypervolume Contribution
- Select extremum points with probability $p_{\text {extreme }}$
- Otherwise select parent $i$ with probability

$$
p_{i}=\frac{\delta \operatorname{Vol}_{S}\left(f\left(x_{i}\right)\right)^{\alpha}}{\sum_{j} \delta \operatorname{Vol}_{S}\left(f\left(x_{j}\right)\right)^{\alpha}}
$$

## Crossover

- $C_{i}$ Covariance matrix of parent
- $i-1, i+1$ neighbours of the parent on the front in $f$-value
- Covariance matrix of offspring

$$
\begin{aligned}
& C=\left(1-c_{r}\right) C_{i}+\frac{c_{r}}{2}\left(\frac{x_{i-1}-x_{i}}{\sigma_{i}}\right)\left(\frac{x_{i-1}-x_{i}}{\sigma_{i}}\right)^{\top} \\
&+\frac{c_{r}}{2}\left(\frac{x_{i+1}-x_{i}}{\sigma_{i}}\right)\left(\frac{x_{i+1}-x_{i}}{\sigma_{i}}\right)^{\top}
\end{aligned}
$$

## Covariance-Matrix-Adaptation

- Parent $\left(C_{i}, \sigma_{i}, x_{i}\right)$, Offspring ( $C, \sigma, x$ )
- Adapt Covariance matrix of offspring by

$$
C \leftarrow\left(1-c_{\mathrm{cov}}\right) C+c_{\mathrm{cov}}\left(\frac{x-x_{i}}{\sigma_{i}}\right)\left(\frac{x-x_{i}}{\sigma_{i}}\right)^{\top} .
$$

- Same for parent


## Step Size adaptation

- Success based as in MO-CMA-ES
- Running estimate of success rate
- Adjust $\sigma$ until success rate $1 / 2$


## Multi-Objective Exploration

- Dominance-based selection gets stuck in local optima
- Run $k=100$ instances in round robin fashion
- $D$ initial points per instance
- Merge fronts after $10^{4} \mathrm{D}$ iterations
- Run single front until budget exhausted


## Results on Sphere/Sphere



## Results on Sphere/Rastrigin



## Results on Sphere/Rastrigin



## Overall Results



Thanks

## See you at BBComp Session!

