Hypervolume-based DIRECT

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DIRECT (DIving RECTangles) Framework

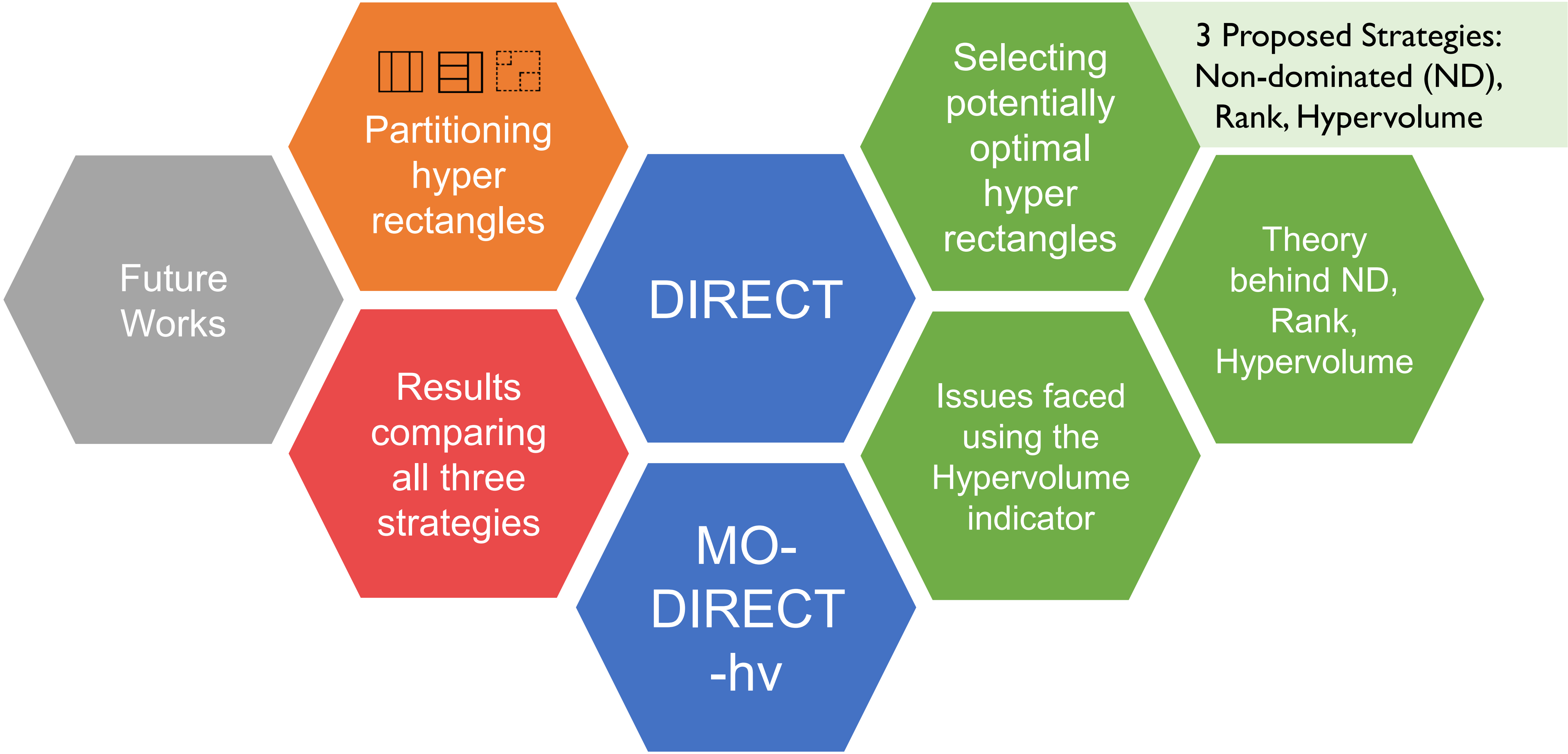
Splits the solution space in the form of a hyperrectangle into smaller hyperrectangles

Partitioning hyper rectangles

DIRECT

Selecting potentially optimal hyper rectangles

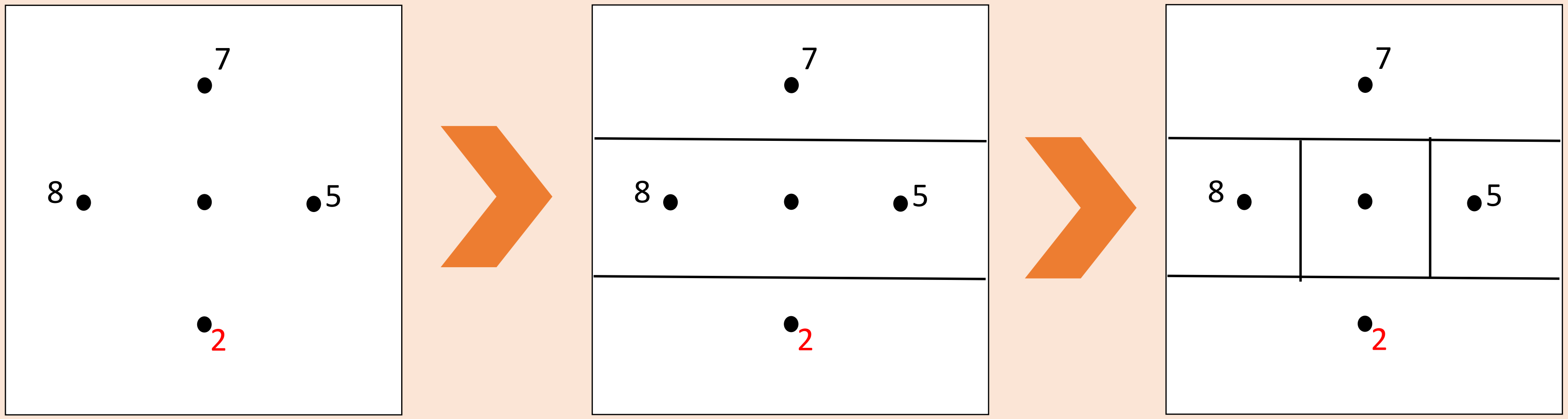
Using solution quality indicators and the size of the hyper rectangles to identify potentially optimal areas in solution space


Future Works
Partitioning hyper rectangles
Results comparing all three strategies
DIRECT
MO-DIRECT -hv
Selecting potentially optimal hyper rectangles
3 Proposed Strategies: Non-dominated (ND), Rank, Hypervolume
Issues faced using the Hypervolume indicator
Theory behind ND, Rank, Hypervolume

Partitioning hyper rectangles

Basic Rule: Start partitioning with the best objective function value
Rationale: They will be bigger size rectangles, hence more likely to be chosen as potentially optimal hyperrectangles

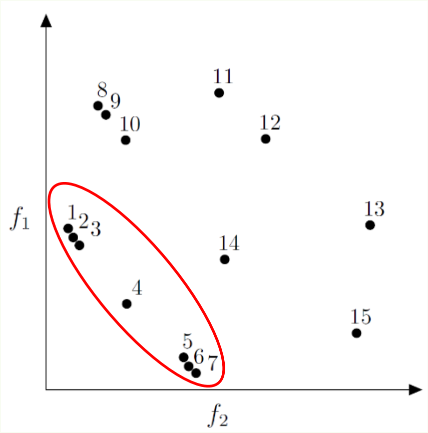
2D example



Non-dominated (ND)

Selecting hyperrectangles based on

- Non-dominated solutions
- Size of hyperrectangles

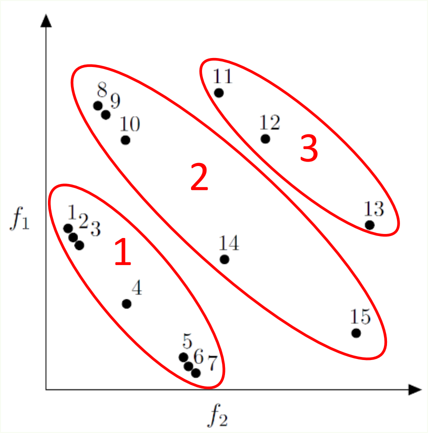


Selecting potentially optimal hyper rectangles

Rank

Selecting hyperrectangles based on

- Rank (Different fronts)
- Size of hyperrectangles



Hypervolume (HV)

Selecting hyperrectangles based on

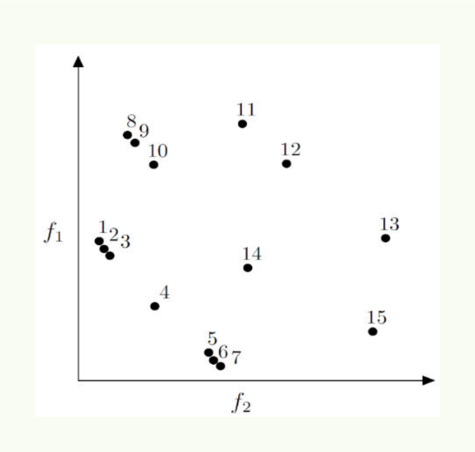
- Hypervolume
- Size of hyperrectangles

Hypervolume:
Quality indicator of non-dominated solutions
Measures the diversity of the solution through the area of space it occupies

Non-dominated solutions

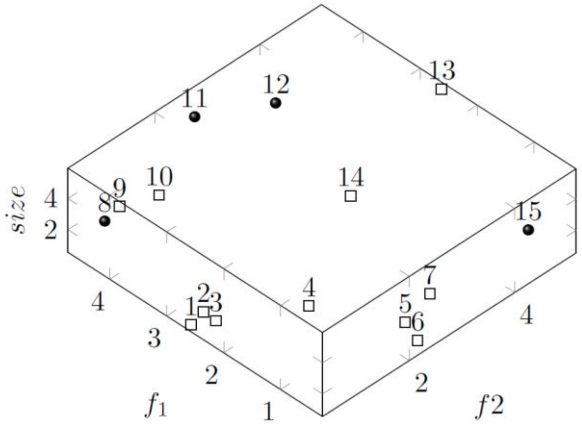
ND: 1,2,3,4,5,6,7
Rank: 7 (largest hyperrectangle)
HV: 4 and 7 (4 has large hypervolume because lack of other non-dominated solutions around it)

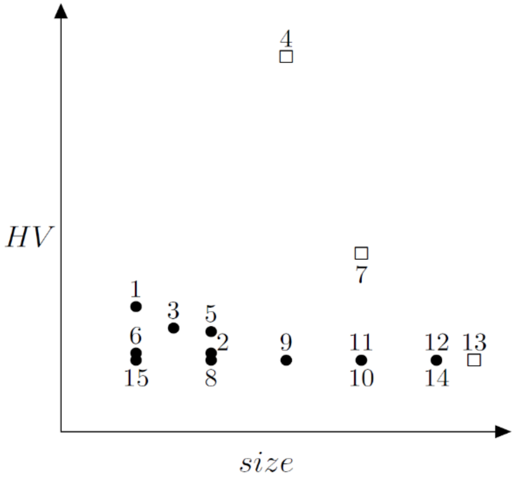
Using hypervolume to select potentially optimal hyperrectangles allows careful selection on the non-dominated front

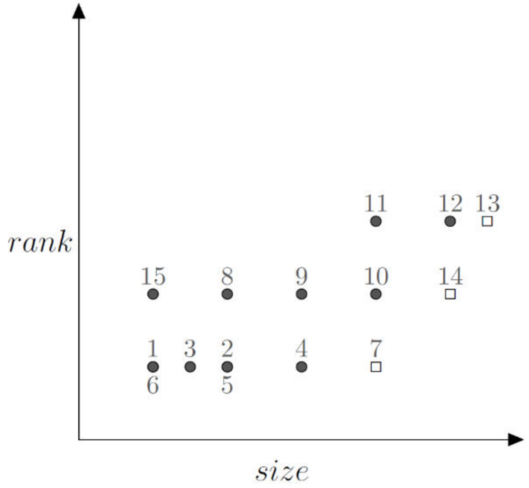


- ❑ Chosen as potentially optimal point
- ● Not chosen as potentially optimal point

ND strategy	Hypervolume	Rank

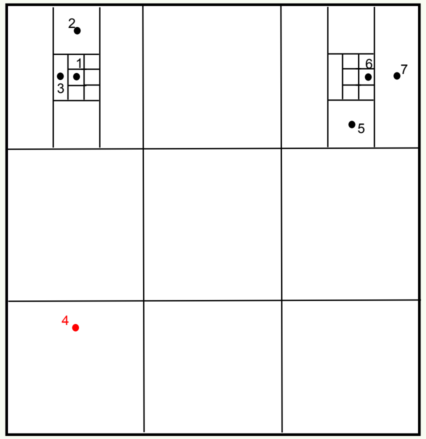


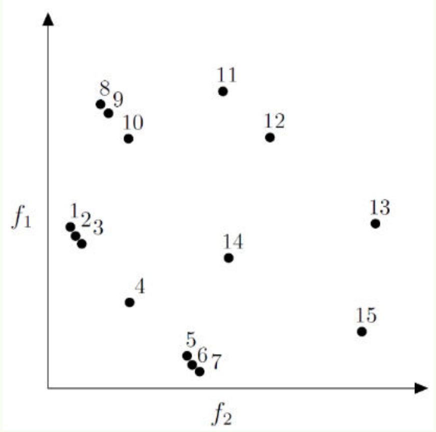




Issues faced using the Hypervolume indicator

Consider…





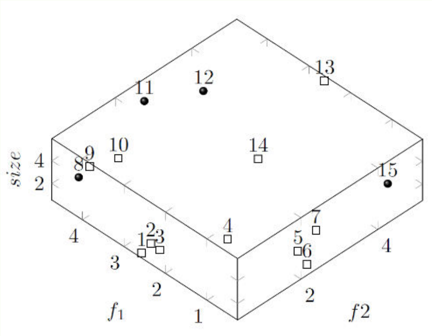
If point 4 is not yet discovered, hypervolume strategy will continue searching the top left and right areas

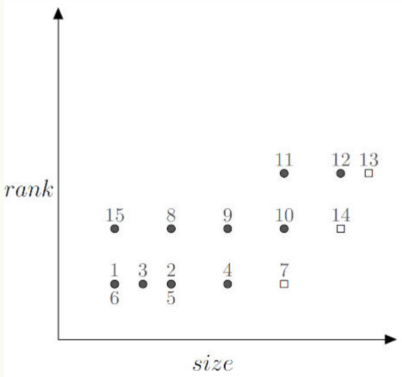
→ Stuck in the local optima

Need to search unexplored areas

Rank Strategy used to explore unexplored space

Consider other strategies…

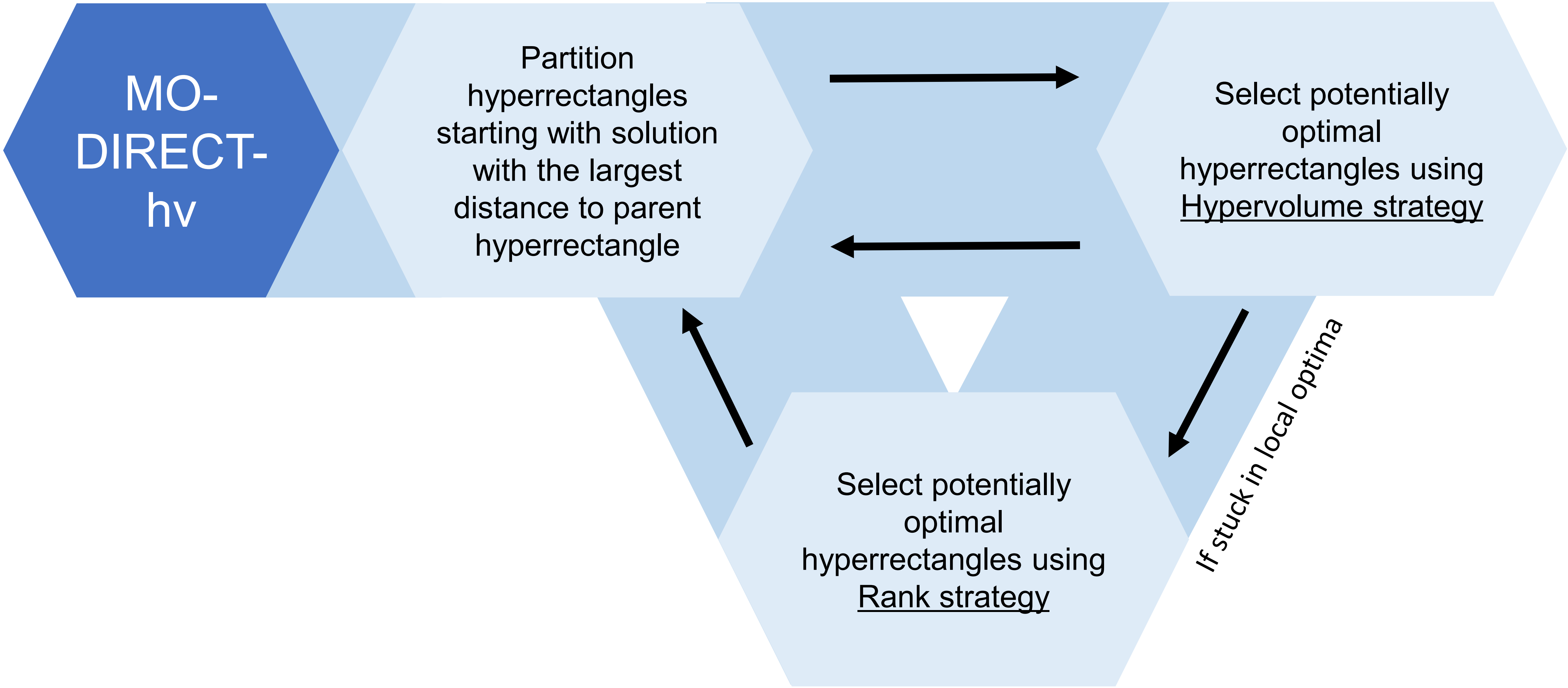


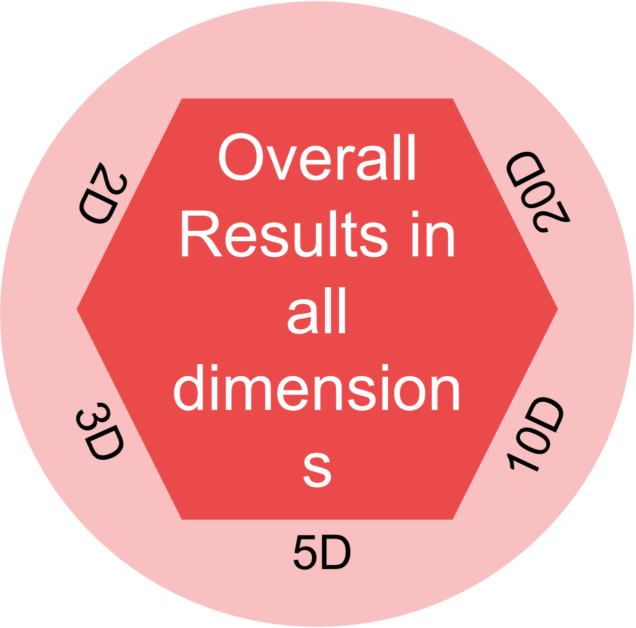


ND: 11 points, 7 non-dominated
Rank: 3 points, 1 non-dominated

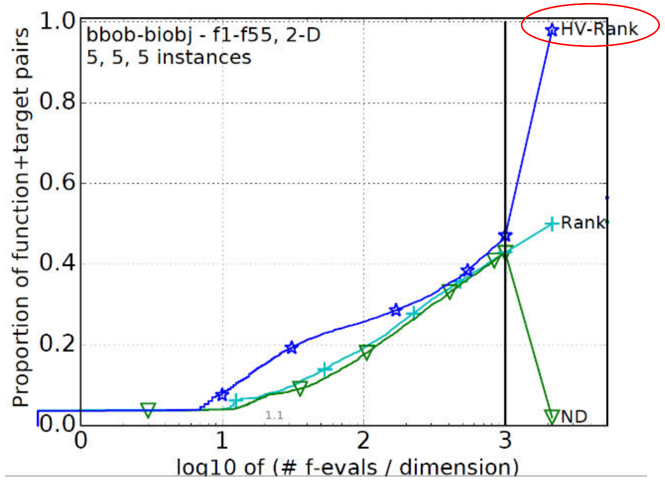
Since, we want to explore unexplored areas not covered by non-dominated solutions,
Rank is a better choice.

Need to search unexplored areas

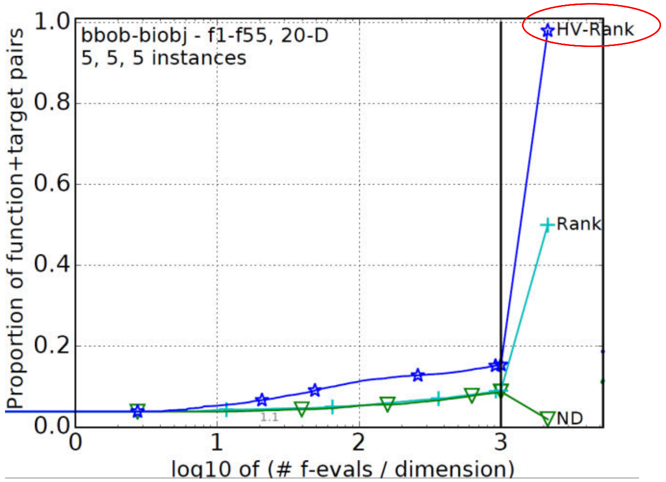

MO-DIRECT-hv
Partition hyperrectangles starting with solution with the largest distance to parent hyperrectangle
Select potentially optimal hyperrectangles using Hypervolume strategy
Select potentially optimal hyperrectangles using Rank strategy
If stuck in local optima



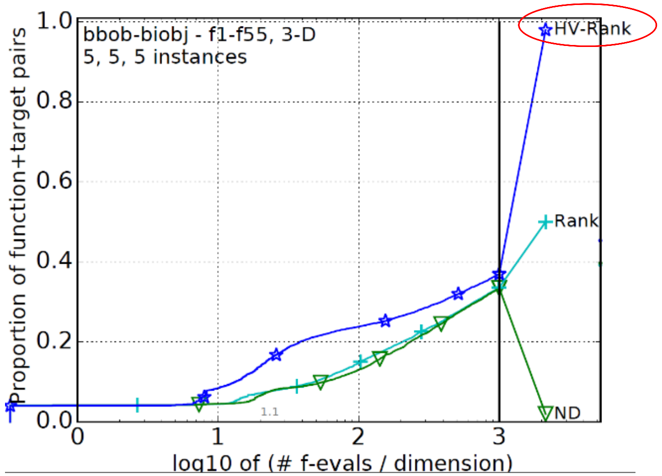
Overall Results in all dimension s
2D
20D
3D
10D
5D



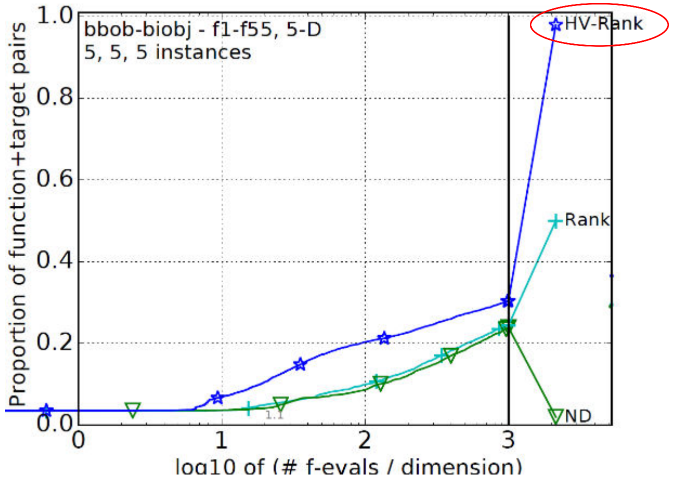
bbob-biobj - f1-f55, 2-D
5, 5, 5 instances
HV-Rank
Rank
ND
Proportion of function+target pairs
log10 of (# f-evals / dimension)



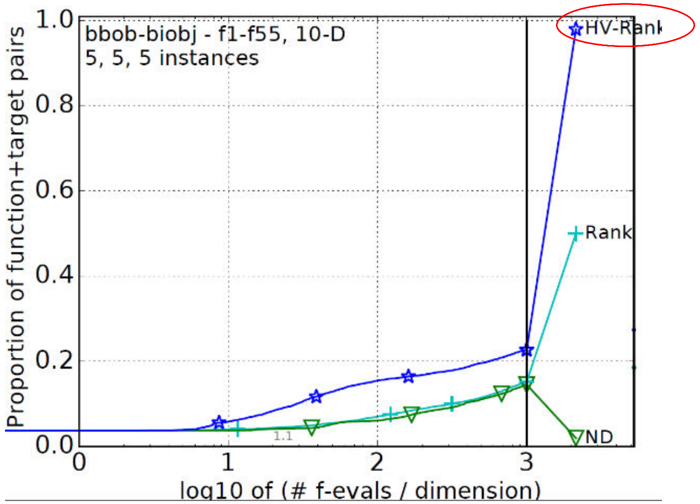
bbob-biobj - f1-f55, 20-D
5, 5, 5 instances
HV-Rank
Rank
ND
Proportion of function+target pairs
log10 of (# f-evals / dimension)



bbob-biobj - f1-f55, 3-D
5, 5, 5 instances
HV-Rank
Rank
ND
Proportion of function+target pairs
log10 of (# f-evals / dimension)



bbob-biobj - f1-f55, 5-D
5, 5, 5 instances
HV-Rank
Rank
ND
Proportion of function+target pairs
log10 of (# f-evals / dimension)



bbob-biobj - f1-f55, 10-D
5, 5, 5 instances
HV-Rank
Rank
ND
Proportion of function+target pairs
log10 of (# f-evals / dimension)

Main Findings

In high dimensions, results start to form a plateau

Excessive sampling due to partitioning strategy

1 potentially optimal point = 2 * 20 sampled points in a 20D problem

MO-DIRECT-hv performs better in non multi-modal problems

Validates and reflects the proposed idea of MO-DIRECT-hv getting stuck in local minima

MO-DIRECT-hv outperforms ND and Rank strategies

Future Works

New Partitioning Strategies to reduce sampled points at high dimensions

Tuning of parameters to determine if the algorithm is stuck in local optima

Testing MO-DIRECT-hv on multiobjective benchmark problems